Precision Measurement of Inter-Facet Angles on Faceted Gems Using a Goniometer

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A classic two-circle reflecting goniometer was used to measure inter-facet angles on five faceted diamonds that included round brilliants and fancy shapes. The instrument provided significantly better precision (to within 2 minutes, or 0.034°) than the non-contact optical scanner that is customarily used at GIA for this purpose. With some procedural modifications, the goniometer could make measurements of all inter-facet angles, including the pavilion facets. The technique is potentially valuable for producing a well characterized set of reference stones for calibrating non-contact optical scanners.

In the modern gem trade, dimensions and facet angles on polished diamonds (figure 1) are usually measured with a computerized non-contact optical scanner (see Reinitz et al., 2005). Such devices are used by gem laboratories as part of the procedure for grading diamonds, and large manufacturers also employ them to determine the most profitable cuts. The scanner typically consists of a high-resolution digital camera, a rotating stage, a light source, and associated software. The camera takes hundreds of profile images as the diamond (usually placed table-down) rotates on the stage. The software then generates a 3D model of the polished diamond and calculates values for the dimensions, proportions, facet angles, and facet positions. The process may take as little as 10 seconds, depending on the number of pictures taken.

Makers of non-contact optical scanners usually claim a linear accuracy of ~10 µm and an angular accuracy of ~0.1°. But each manufacturer uses somewhat different algorithms in their proprietary software to generate the final 3D model, so the resulting values of the dimensions and angles can deviate from one maker to another. In fact, the results may vary from instrument to instrument. Therefore, users should establish a master set of standards, in the form of faceted gemstones with known dimensions and angles, so they can check the instrument to ensure accuracy and repeatability for daily operation. Unfortunately, these standard sets are not readily available. In addition, for calibration purposes the angles and dimensions of these "master stones" must be measured to even higher precision than non-contact optical instrumentation can provide. In this study, we examine the feasibility of using a well-established optical instrument—a classical two-circle reflecting goniometer—to measure the angles on faceted diamonds to very high precision, without relying on image analysis and computer algorithms.

BACKGROUND

For this investigation, we chose Cornell University's two-circle reflecting goniometer (figure 2). This type of goniometer was used extensively by mineralogists in the late 1800s and early 1900s to study the angles between faces on natural crystals (Burchard, 1998). Before the advent of X-ray diffraction techniques, these instruments played a major role in obtaining fundamental measurements that provided a deeper understanding of the geometry and structure of crystals.

While goniometers range in complexity and date back as early as the 1700s, the version used in this investigation is named for its inventor, Prof. V. M. Goldschmidt, the famous crystallographer and author of *Atlas der Krystalformen*. Made by Stoe & Rheinheimer of Heidelberg, with whom Goldschmidt worked closely, the Cornell instrument is a Model A, circa 1920 (O. Medenbach, pers. comm., 2011). According to Burchard (1998, p. 574), there are probably fewer than 800 reflecting goniometers predating World War II in existence, and only 10 of this particular model were manufactured in 1920. Interestingly, this goniometer was inspired by the need to measure a newly discovered gem. The pink spodumene crystals identified by George F. Kunz and now known as kunzite were so large that they could not fit in the pre-1905 Goldschmidt models

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Figure 1. These diamonds were used for the facet angle measurements in this investigation. From left to right, the stones weigh 0.20, 0.86, 0.62, 0.71, and 0.40 ct. Photo by Robert Weldon.

(Burchard, 1998). This design innovation suited our present needs in that it afforded adequate space for specialized mounting of the diamond. Modern reflecting goniometers (e.g., the Huber 302 model) employing the same principles as the Cornell instrument are typically used to orient crystals for X-ray diffraction. These high-precision instruments are capable of the same level of accuracy as the Cornell goniometer.

INSTRUMENTS AND METHODS

The two-circle reflecting goniometer consists of several key components, including two *wheels* (or *circles*), a

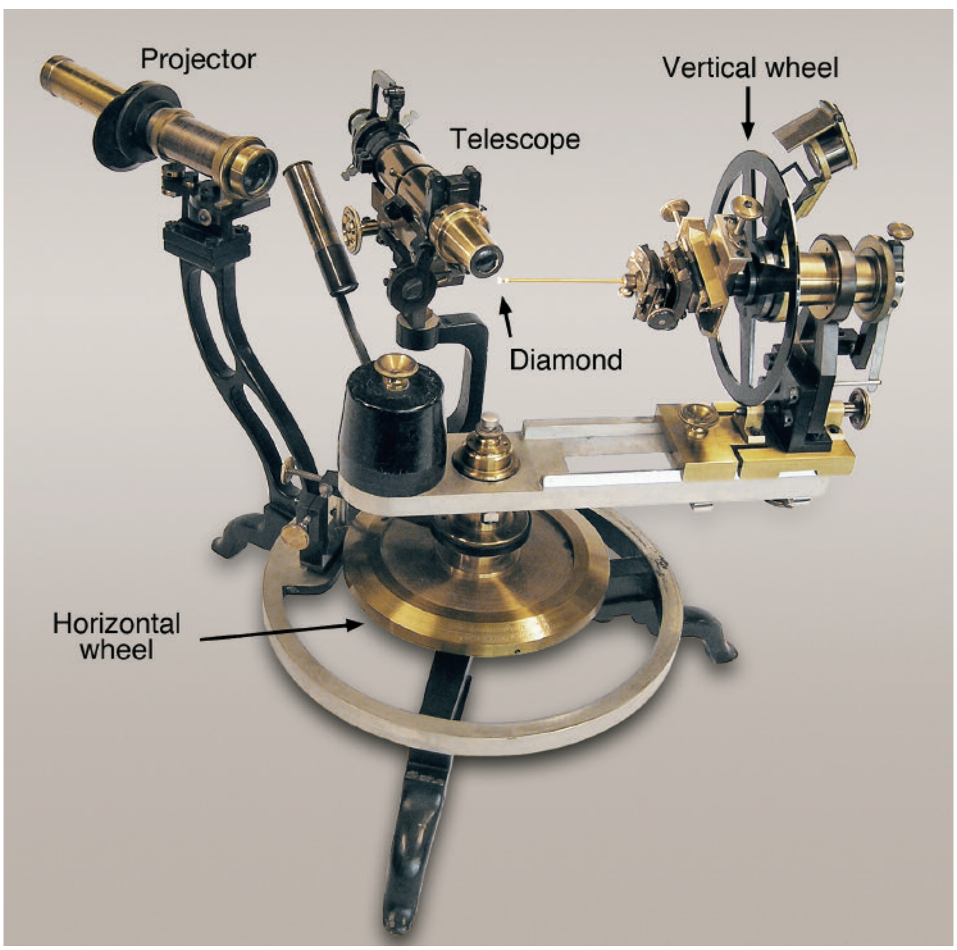

Figure 2. Cornell University's classic two-circle reflecting goniometer, built circa 1920, was used in this study. The instrument is approximately 42 cm tall. Photo by E. A. Skalwold.

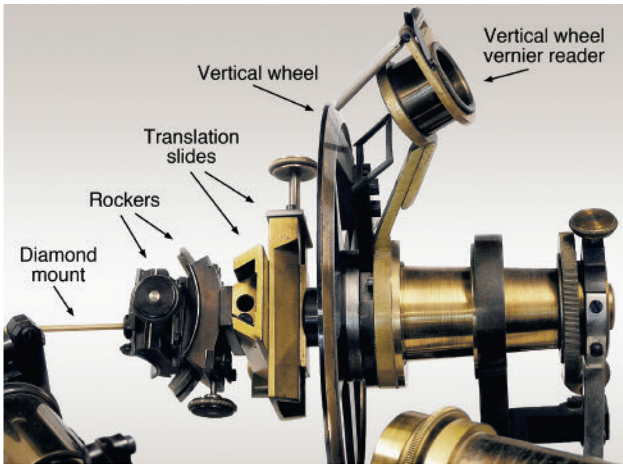

Figure 3. The goniometer head contains the rockers and translation slides needed to orient and position the specimen. After the gem was positioned at the end of the brass mounting tube, the rockers were used to make small corrections. Photo by E. A. Skalwold.

light source (*collimator* or *projector*), and a *telescope* (figures 3–5). The goniometer *head* features *slides* for centering the specimen at the point where the axes of the two wheels intersect so it can be precisely rotated to nearly any orientation. The goniometer head also has *rockers* for orienting the specimen. The telescope has an *auxiliary flip-up lens* to switch from focusing on the surface of the specimen to focusing on the target, a *Maltese cross* in the light source (see figure 4). Using the auxiliary lens, the operator can look directly at the specimen and observe flashes of light reflected from its facets, while rotating it through all possible orientations by turning the two wheels. Once a reflection has been spotted, the auxiliary lens is removed so the telescope is focused on the Maltese cross, using the specimen's reflecting face as a mirror. This causes the Maltese cross to appear in the telescope. Once the specimen has been oriented so that the Maltese cross image is centered on the crosshairs of the telescope, a unique orientation of the reflecting facet has been established. The operator can then record the angles on the scales of the two wheels, henceforth referred to as *angular coordinates* (box A), and then look for a new reflecting facet. When the image of the Maltese cross is exactly centered on the crosshairs, this new facet

In Brief

- Non-contact optical scanners are important components for grading diamonds in gem laboratories.
- Scanners are claimed to have a precision of ~0.1° for facet angle measurements, compared to a precision of 0.034° for the goniometer used in this study.
- Although time consuming, goniometer measurements of facet angles are useful for highly precise applications such as producing reference stones for calibrating optical scanners.

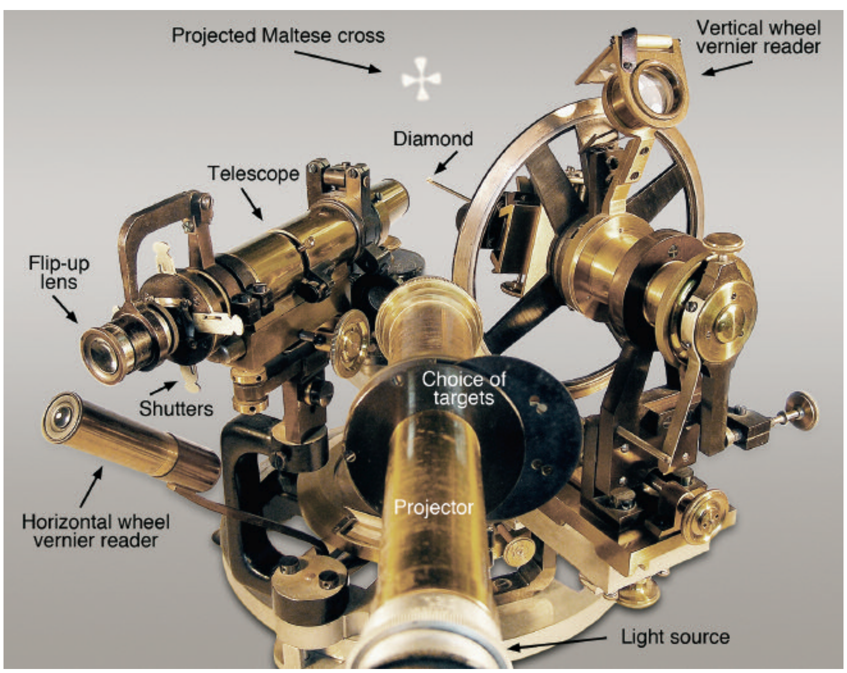

Figure 4. The Maltese cross projected onto the far surface is one of several target figures that can be chosen with the projector (collimator) of the goniometer. The position of the reflected target observed in the telescope is very sensitive to the orientation of the facet being observed. When the target image is perfectly centered on the telescope crosshairs, the angular coordinates of the facet can be measured very accurately. The angle between the projector and the telescope was reduced as far as possible without blocking the light path, enabling measurement of the pavilion facets. Photo by E. A. Skalwold.

BOX A: DERIVATION OF INTER-FACET ANGLES

The underlying principle of the technique used in this study is spherical geometry. If we place a round brilliant-cut diamond in the center of an imaginary sphere, the normals—that is, the imaginary lines perpendicular to the facets—will intersect the sphere at unique locations (designated by x in figure A-1). The *angular coordinates* of these locations provide all of the information needed to determine the stone's inter-facet angles.

The reflecting two-circle goniometer measures a pair of angular coordinates for each facet: ρ (the zenith angle, measured by the horizontal wheel) and ϕ (the azimuthal angle, measured by the vertical wheel). The angular coordinates of the table facet are determined first, as they provide the reference coordinates for all other measurements. The angular coordinates of a different facet (for instance, a crown or star facet) are obtained by subtracting the table facet's coordinates from the new readings. The angles between two facets can be calculated from the coordinates using the equation:

$$\alpha = \cos^{-1}(\sin\rho_1 \sin\phi_1 \sin\rho_2 \sin\phi_2 + \sin\rho_1 \cos\phi_1 \sin\rho_2 \cos\phi_2 + \cos\rho_1 \cos\rho_2)$$

where (ρ_1, ϕ_1) and (ρ_2, ϕ_2) are the angular coordinates of the normals to the two facets. This equation can be further reduced if one of the facets is the table facet, which has the angular coordinates of (0, 0). Setting (ρ_1, ϕ_1) to (0, 0), the equation is reduced to:

$$\alpha = \cos^{-1}(\cos 0 \cos\rho_2)$$

and because $\cos 0 = 1$, the result is $\alpha = \rho_2$.

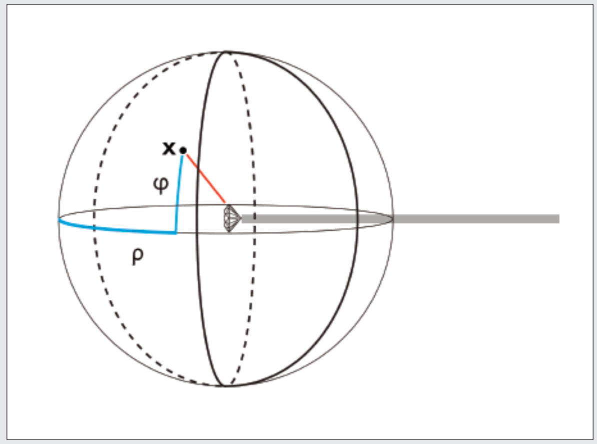

Figure A-1. A line perpendicular to a facet on a round brilliant intersects an imaginary sphere (point x) at angular coordinates ρ (zenith angle) and ϕ (azimuthal angle).

has the same orientation with respect to the telescope and light source. Reading the scales provides angular coordinates, from which the angles between observed facets can be calculated. The positions of the light source and the observation telescope must be kept fixed throughout all measurements of a particular stone.

In the process of finding reflections from a faceted stone, a constant challenge is created by multiple reflections produced by the many facets on the diamond. The *shutters* shown in figures 4 and 5 solve this problem by limiting the field of view to just that portion of the light reflected from the facet of interest. Once the selection is made, only the Maltese cross produced by that facet will appear.

The main challenge of using reflecting goniometers for measuring angles on faceted stones is accessing the pavilion facets on some fancy cuts such as oval and pear shapes. This is because the light source and telescope lie in the same plane as one of the circles used for making measurements, thus limiting the range of angles that can be measured with that wheel. Therefore, we mounted the diamond at the end of a long (76 mm) brass tube, which allowed us to move the vertical wheel and the goniometer head outward. This significantly reduced the angle between the light source and the telescope without obstructing the light path. The first step was to mount a round brilliant-cut diamond (~0.25 ct) so the table facet could be oriented perpendicular to the axis of the vertical wheel. A high-precision drill press was used to orient the table facet perpendicular to the axis of the brass tube. Once the stone was cemented in place, only minor adjustments using the rockers and slides were necessary. This arrangement made it possible to obtain reflections from all the facets on the stone without the need to remount it.

After positioning the table facet so it was clearly visible in the telescope, we oriented it to reflect light from the collimator into the telescope. By flipping up the auxiliary lens, we could use the Maltese cross to refine the orientation of the table facet by centering its reflected image on the crosshairs. Then the specimen was rotated about the axis of the horizontal wheel until

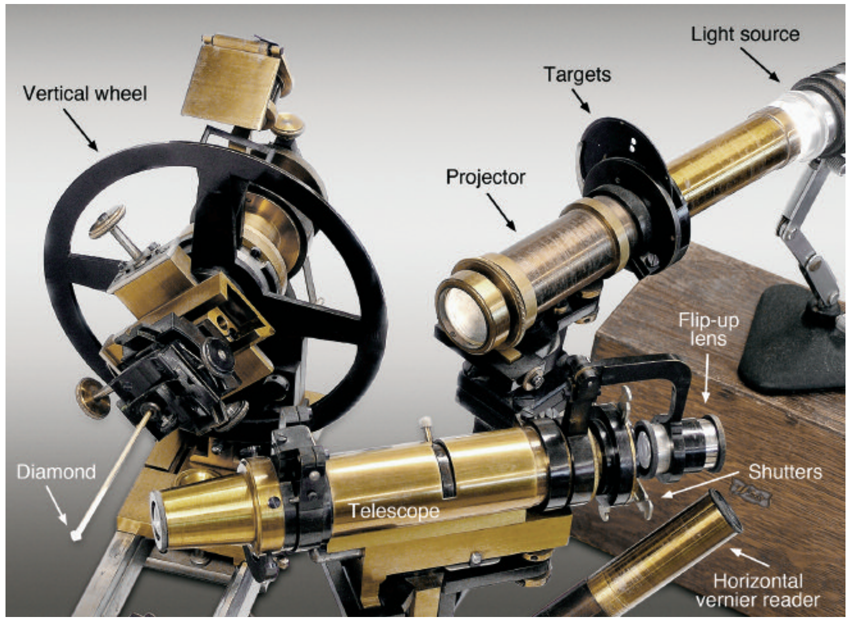

Figure 5. This view of the goniometer shows the positioning of the diamond, light source, and telescope for obtaining reflections from the pavilion facets. Photo by E. A. Skalwold.

the pavilion facets were in position to reflect light from the collimator/projector to the telescope (figures 4 and 5). We could then measure the angles between all the facets on a round brilliant. Oval- and pear-cut stones posed more of a challenge, in some cases requiring the operator to aim the light source between the spokes of the vertical wheel. Nevertheless, we were able to obtain measurements on these fancy cuts as well. In instances where the spokes blocked the light beam, we estimated the angle using an averaging method detailed in the Discussion section.

Five faceted diamonds (see table 1) were measured on the goniometer and the non-contact optical scanner: three round brilliants (nos. 1, 2, and 3), one oval brilliant (no. 92), and one pear brilliant (no. 93). With the goniometer, we focused on measuring the bezel facets and pavilion facets since the angles between these facets and the table have the most significant impact on the cut grade (Hemphill et al., 1998; Reinitz et al., 2001; Moses et al., 2004). We also measured the star facets on one of the round brilliants (no. 2), because these facets have the lowest angles from the table and are easily obscured when using a non-contact measuring device. Each diamond was inscribed on the girdle, and this inscription served as a standard position from which to begin recording the measurements. The same diamonds were also measured 10 times on a commercially available non-contact optical scanner, set for total of 400 scans and a scanning rate of 10 scans/second. The scanner is representative of those used in GIA's laboratory since 2010. According to the manufacturer, the precision of the angle measurement is 0.1°.

RESULTS AND DISCUSSION

The calculated inter-facet angles for each diamond are listed in table 2. At first glance, the results from the goniometer compare very favorably to those from the non-contact optical scanner. In more than 78.3% of the tests, the angles measured by each instrument were within 0.2° of one another. Only 40.8% of the measurements achieved less than 0.1° deviation, and six of the 83 measurements deviated significantly (greater than 0.4°, indicated by bold font). The largest deviation, which occurred from a star facet, was 1.28°. To determine the precision of the goniometer measurements, we first obtained readings of the bezel or pavilion facets on the two fancy-shaped diamonds, which, due to their larger size, required shifting the stone using the translation slides. We then returned to the table facet to see

TABLE 1. Diamonds examined in this study.

Sample no.	Shape	Weight (ct)	Dimensions (mm)
1	Round	0.20	3.77–3.80 x 2.30
2	Round	0.40	4.79–4.82 x 2.75
3	Round	0.62	5.66–5.70 x 3.37
92	Oval	0.86	7.43 x 5.19 x 3.24
93	Pear	0.71	7.75 x 4.92 x 3.19

TABLE 2. Comparison of angles (in degrees) relative to the table facet, measured by the goniometer and non-contact optical scanner.[a]

Sample no.	1 (Round)		2 (Round)		3 (Round)		92 (Oval)		93 (Pear)	
Facet	Goniometer	Scanner	Goniometer	Scanner	Goniometer	Scanner	Goniometer	Scanner	Goniometer	Scanner
Bezel 1	34.00	33.8	30.08	29.8	26.58	26.4	34.85	34.9	35.77	35.9
Bezel 2	34.00	33.9	30.10	29.9	26.67	26.5	36.92	36.8	38.23	38.3
Bezel 3	33.97	34.1	30.03	30.0	26.53	26.6	38.17	38.1	32.60	32.6
Bezel 4	33.98	34.2	29.93	30.1	26.63	26.6	35.77	35.7	31.22	31.2
Bezel 5	33.97	34.2	29.92	30.1	26.58	26.6	35.53	35.5	33.03	**32.5**
Bezel 6	34.02	34.0	29.92	29.9	26.65	26.6	37.33	37.4	34.83	34.7
Bezel 7	34.02	33.8	29.93	29.6	26.67	26.5	37.73	37.8	41.02	41.2
Bezel 8	34.05	33.8	30.07	29.7	26.58	26.4	35.80	36.0	–	–
Bezel 1[b]	–	–	–	–	–	–	34.87	34.9	35.77	35.9
Pavilion 1	40.52	40.6	40.40	40.6	41.88	42.0	36.87	36.5	38.53	38.4
Pavilion 2	40.40	**40.8**	40.38	40.6	41.92	41.9	36.32	36.6	41.52	41.4
Pavilion 3	40.57	**41.0**	40.43	40.5	42.03	41.9	36.23	**37.0**	41.37	41.4
Pavilion 4	40.92	41.0	40.67	40.5	42.03	41.9	36.67	36.7	36.90	36.7
Pavilion 5	41.08	40.9	40.72	40.5	42.05	42.0	–	–	36.93	36.9
Pavilion 6	41.02	40.7	40.67	40.5	41.97	42.1	–	–	40.83	40.7
Pavilion 7	40.83	40.6	40.47	40.5	41.68	**42.1**	–	–	–	–
Pavilion 8	40.45	40.6	40.42	40.5	42.05	42.1	–	–	–	–
Star 1	–	–	15.47	15.4	–	–	–	–	–	–
Star 2	–	–	15.50	15.5	–	–	–	–	–	–
Star 3	–	–	15.52	**16.8**	–	–	–	–	–	–
Star 4	–	–	15.40	15.6	–	–	–	–	–	–
Star 5	–	–	15.40	15.7	–	–	–	–	–	–
Star 6	–	–	15.38	15.5	–	–	–	–	–	–
Star 7	–	–	15.42	15.5	–	–	–	–	–	–

[a] *Bold font indicates a deviation of greater than 0.4° between measurements.*
[b] *This bezel 1 measurement was performed after a full 360° rotation of the vertical wheel for purpose of assessing the instrument's precision.*

if the readings of the table reference point had changed. The maximum deviation was 2 minutes (0.034º), which we established as representing the precision of the instrument. The finest division on the goniometer scale is 1 minute or 0.017º. According to the historical literature (e.g., Tutton, 1922), the two-circle reflecting goniometer was believed to be capable of a precision of 30 seconds (0.0083º). With careful estimation, one can easily estimate the angles between two divisions and improve the precision down to 30 seconds. But even at a precision of 2 minutes, we were achieving far better precision than the non-contact measuring device (for which 10 measurements of each stone showed a repeatability of within 0.1º).

The main source of uncertainty in goniometer measurements is human error in reading the scales. During this investigation, two observers read each measurement while a third independently checked their readings. The other source of error occurs when the Maltese cross image is blocked by part of the instrument, as described above. We encountered this situation only a couple of times in the course of our research, and obtained an average of two measurements to overcome this problem. The first measurement was taken at the nearest horizontal position where the full Maltese cross was visible, and the second was taken from a different position with approximately the same separation between the center of the crosshairs and the first Maltese cross position. The typical separation between these two positions was within 10 minutes (0.17º). This approach provided consistent results, and the error introduced should still be quite comparable to the direct measurement (i.e., <2 minutes, or 0.034º).

As described earlier, non-contact optical scanners rely on computer algorithms to construct a 3D model of the actual stone. These algorithms require a basic model of the stone—an ideal plot of a round brilliant cut, emerald step cut, and so forth—to achieve high precision. In most cases, this requires the operator to choose from a selection of models available in the software. For the sake of argument, we will disregard the possibility of choosing the wrong model. More likely, the actual stone has an extra facet, or foreign material such as dirt or lint on its surface during measurement. Surface contamination is most problematic for determining the low angles of star facets on a brilliant cut, because the shadows can badly skew the image analysis. Indeed, the largest deviation we observed (1.28º) was from a star facet. Also challenging are some fancy-shape measurements, such as crown facets on an emerald cut that are at a low angle from the table facet.

Although the two-circle reflecting goniometer avoids the use of computer algorithms or pre-installed ideal models of faceted gems, its application in a modern gem lab will be limited because it is time consuming. Nevertheless, its use may be justified in certain circumstances. For example, its high precision make it an excellent technique to establish a master set of reference stones with very accurately determined inter-facet angles. Much like the master sets used for color-grading diamonds, these can be considered calibration standards for angle measurement. We believe the deviations shown in table 2 arise mostly from 3D model construction or some surface contamination. In the gem lab these discrepancies can potentially result in different cut grades, since the cut-grade software uses whatever pavilion and crown angles are obtained from the non-contact optical scanner. To ensure the accuracy and precision of these measurements, optical scanners need to be checked routinely with master stones.

Another possible use for the two-circle reflecting goniometer is evaluating facet quality. The small curvature shown by a poorly cut facet is easily observed via deformation or blurring of the reflected Maltese cross target. This effect was not observed in any of the facets we measured in this study.

CONCLUSIONS

The two-circle reflecting goniometer can be used to measure inter-facet angles on faceted gemstones with a very high degree of precision (to within 2 minutes, or 0.034º). The angular coordinates of any facet can be determined without remounting the sample. The instrument can provide a valuable means to independently calibrate the non-contact optical scanners widely used in gem labs, and can also be used to evaluate facet quality. Finally, the classic goniometer provides an excellent basis for the future design of a fully automated optical goniometer made specifically for faceted gemstones.

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